Foundation of direct perturbation method for dark solitons

1999 J. Phys. A: Math. Gen. 32 3939

(http://iopscience.iop.org/0305-4470/32/21/307)

J. Phys. A: Math. Gen. **32** (1999) 3939–3945. Printed in the UK PII: S0305-4470(99)99755-X

Foundation of direct perturbation method for dark solitons

Nian-Niang Huang†§, Sien Chi† and Xiang-Jun Chen‡

† Institute of Electro-Optical Engineering, National Chiao Tung University, Hsinchu, Taiwan 30049, Republic of China

‡ Department of Physics, Jinan University, Guangzhou 510063, People's Republic of China

Received 27 November 1998, in final form 25 February 1999

Abstract. The foundation of the direct perturbation theory for solitons is a complete set of the squared Jost functions. With a suitable definition of the adjoint functions and inner products which yields orthogonal relations, the expansion of the unity is obtained and the completeness is shown by a generalized Marchenko equation. The direct perturbation method for dark solitons is generalized to the multi-soliton case.

1. Introduction

Since optical dark solitons have been theoretically predicted and experimentally realized, it has become important to study how these dark solitons are affected by various perturbations [1–3]. The basic difficulty one will encounter during the development of a perturbation method for dark solitons is that the background of dark solitons, i.e. the nonvanishing boundary values, are well known to be dependent on time when perturbations are added in. This property has led to essential difficulties when generalizing the usual perturbation method based on the inverse scattering transform for bright solitons [4–7] to the case of dark solitons. Despite these difficulties, various perturbation methods for dark solitons have been proposed [3, 8–14]. These methods cannot completely overcome those difficulties caused by the background wave and thus cannot be regarded as rigorous methods. However, it should be mentioned that in a method using conservation laws (see [11] and later [12]), the varying background was considered with a trick treating the so-called vanishing perturbations and nonvanishing perturbations separately by assuming that the former do not affect the background. While for the latter ones, they tried to remove the varying background and determined its evolution with an individual equation. It has been shown that the attempt to remove the background is not only unnecessary but also inadequate because of the interrelation (see equation (2) below) among the background and the parameters of the dark solitons [16].

Recently, a rigorous direct perturbation theory for the one-soliton case was developed [16], based upon a complete set of the squared Jost solutions. Both of the vanishing perturbations and nonvanishing perturbations can be treated on the same foundation. In applications of the rigorous theory [16] to specific problems of temporal dark solitons [17–19] and spatial dark solitons [20], it turns out that the method using conservation laws [11, 12] can still give some correct final results except for the shift of the soliton centre, in spite of its difficulties in self-consistency [13]. It has been pointed out that the method using conservation laws can only yield limited information of perturbations (see, e.g., [2]).

§ Permanent address: Department of Physics, Wuhan University, Wuhan 430072, People's Republic of China.

0305-4470/99/213939+07$19.50

Further development of the direct perturbation theory is natural to generalize it to the multi-soliton case. The key is to establish a complete set of the squared Jost solutions. For bright soliton case the completeness was proved using the assumption of compact support [15], which is no longer valid for dark solitons. That is why in [16] the proof for the completeness had to use explicit expressions of the squared Jost solutions for the one-soliton case. In the present work a generalized Marchenko equation similar to that for the Korteweg–de Vries (KdV) equation [21–23] is derived for the nonlinear Schrödinger (NLS) equation of dark solitons (NLS^+ for short), and the completeness of the squared Jost solutions is proved with it. A general mathematical formalism of the direct perturbation method for dark solitons is then developed.

2. The perturbed equation

The perturbed NLS^+ equation can be written as [5, 16]

$$\mathrm{i}v_t - v_{xx} + 2(|v|^2 - \rho^2)v = \epsilon r[v] \tag{1}$$

where ρ is a positive constant, ϵ is a small parameter and $r[v]$ is a functional of v. When $\epsilon \to 0$, the unperturbed NLS^+ equation can be solved with the boundary condition $v \to \rho$ as $x \to +\infty$ and $v \to \rho\mathrm{e}^{\mathrm{i}\alpha}$ as $x \to -\infty$, where α is a real constant. Within the framework of the inverse scattering transform it can be shown that the poles of the transmission coefficient $a(\zeta)$ in the complex ζ-plane, ζ_n, $n = 1, 2, \ldots, N$, are related to ρ and α by

$$\zeta_n = \rho\mathrm{e}^{\mathrm{i}\beta_n} \qquad \alpha = -\tfrac{1}{2}\sum_{n=1}^{N}\beta_n \qquad 0 \leqslant \beta_n < \pi. \tag{2}$$

Now the problem is to find a perturbed solution with the initial condition $v(x, t = 0) = u(x, t = 0)$, where $u(x, t)$ is usually an exact dark soliton solution of the unperturbed NLS^+ equation. Under perturbations, ζ_n will depend on t to the order of ϵt, and so do ρ and α.

Suppose [15]

$$v = u^a + \epsilon q \tag{3}$$

where u^a is the so-called adiabatic solution which has the same functional form as the exact soliton solution u, but the parameters involved may be ϵt dependent, and ϵq represents the remaining term which is also of the order of ϵ. Substituting (3) into (1), we obtain

$$\mathrm{i}q_t - q_{xx} + 2(2|u|^2 - \rho^2)q + 2u^2\bar{q} = R[u] \tag{4}$$

where

$$R[u] = r[u] - s[u] \qquad s[u] = \frac{1}{\epsilon}\{\mathrm{i}u_t - u_{xx} + 2(|u|^2 - \rho^2)\} = \mathrm{i}u_\tau \tag{5}$$

and $\tau = \epsilon t$. Since (4) is an equation of the order of ϵ (the factor of ϵ has been dropped), the u on the left-hand side and in $r[u]$ is an exact solution of the unperturbed equation, while u in $s[u]$ is the adiabatic solution because $s[u]$ includes derivatives of the discrete spectrum with respect to the slow time τ (the superscript a has been dropped). Equation (4) and its complex conjugate can be written as

$$\left\{\mathrm{i}\partial_t - \boldsymbol{L}(u)\right\}\boldsymbol{q} = \boldsymbol{R} \tag{6}$$

where

$$\boldsymbol{L}(u) = \begin{pmatrix} \partial_{xx} - 2(2|u|^2 - \rho^2) & -2u^2 \\ 2\bar{u}^2 & -\partial_{xx} + 2(2|u|^2 - \rho^2) \end{pmatrix} \tag{7}$$

and $\boldsymbol{q} = (q \;\; \bar{q})^T$, $\boldsymbol{R} = (R \;\; -\bar{R})^T$. Now, the initial condition becomes $\boldsymbol{q}(x, t = 0) = (0 \;\; 0)^T$.

3. Squared Jost functions

By using the Lax equations, we obtain, for example,

$$\left\{\mathrm{i}\partial_t - \boldsymbol{L}(u)\right\}\Psi(x,\zeta) = -4\kappa\lambda\Psi(x,\zeta). \tag{8}$$

It is obvious that the Jost functions and their corresponding squared Jost functions have the same analytical properties. For example, both $\psi(x,\zeta)$ and $\Psi(x,\zeta)$ are analytical on the upper-half plane of ζ. Since the Lax equations are invariant with respect to the reduction transformation [5, 16], $\zeta \to \rho^2\zeta^{-1}$, it can be shown that $\psi(x,\rho^2\zeta^{-1}) = -\mathrm{i}\rho^{-1}\zeta\tilde{\psi}(x,\zeta)$, and hence $\Psi(x,\rho^2\zeta^{-1}) = -\rho^{-2}\zeta^2\tilde{\Psi}(x,\zeta)$, etc. Furthermore, it can be concluded that $a(\rho^2\zeta^{-1}) = \tilde{a}(\zeta)$ and the zeros of $a(\zeta)$ are simple and satisfy (2). From (8) we have, for example,

$$\left\{\mathrm{i}\partial_t - \boldsymbol{L}(u)\right\}\Psi(x,\zeta_n) = -4\kappa_n\lambda_n\Psi(x,\zeta_n) \tag{9}$$

$$\left\{\mathrm{i}\partial_t - \boldsymbol{L}(u)\right\}\dot{\Psi}(x,\zeta_n) = -4\kappa_n\lambda_n\dot{\Psi}(x,\zeta_n) - 2\left(\zeta_n + \rho^4\zeta_n^{-3}\right)\Psi(x,\zeta_n). \tag{10}$$

4. Adjoint squared Jost functions

It is reasonable to demand that the inner product between the squared Jost function and its adjoint are proportional to a δ function in the continuous spectrum. We define the inner product to be [15]

$$\langle\Psi(\zeta')|\Psi(\zeta)\rangle = \int_{-\infty}^{\infty}\mathrm{d}x\,\Psi(x,\zeta')^A\Psi(x,\zeta) \tag{11}$$

where the adjoint function is given by

$$\Psi(x,\zeta)^A = \Phi(x,\zeta)^T(\mathrm{i}\sigma_2) = \left(-\phi_2^2(x,\zeta)\;\;\phi_1^2(x,\zeta)\right). \tag{12}$$

From the first Lax equation, we have

$$\frac{\mathrm{d}}{\mathrm{d}x}\{W[\phi(x,\zeta'),\psi(x,\zeta)]\}^2 = -\mathrm{i}2(\lambda-\lambda')\Psi(x,\zeta')^A\Psi(x,\zeta) \tag{13}$$

where $W[\cdots]$ is the usual Wronskian. Hence we obtain

$$\int_{-\infty}^{\infty}\mathrm{d}x\,\Psi(x,\zeta')^A\Psi(x,\zeta) = \lim_{L\to\infty}\frac{1}{-\mathrm{i}2(\lambda-\lambda')}\{W[\phi(x,\zeta'),\psi(x,\zeta)]\}^2\big|_{-L}^{L}. \tag{14}$$

Noting that the real ζ and ζ' should be replaced by $\zeta+\mathrm{i}0$ and $\zeta'+\mathrm{i}0$ and the limit is considered as the Cauchy principal value, we obtain

$$\langle\Psi(\zeta')|\Psi(\zeta)\rangle = a(\zeta)^2 2\pi(1-\rho^2\zeta^{-2})\delta(\zeta-\zeta'). \tag{15}$$

Since (13) is valid on the upper half-plane, by applying an operator $\mathrm{d}^2/\mathrm{d}\lambda^2$ to it, taking $\zeta = \zeta' = \zeta_n$ and then integrating, we obtain

$$\langle\Psi(\zeta_m)|\dot{\Psi}(\zeta_n)\rangle = \mathrm{i}\dot{a}(\lambda_n)^2(1-\rho^2\zeta_n^{-2})\delta_{mn} \tag{16}$$

and the same result for $\langle\dot{\Psi}(\zeta_m)|\Psi(\zeta_n)\rangle$.

Applying the operator

$$\left\{\frac{\mathrm{d}^3}{\mathrm{d}\zeta^3} + 3\frac{\mathrm{d}}{\mathrm{d}\zeta'}\frac{\mathrm{d}^2}{\mathrm{d}\zeta^2}\right\}$$

to (13), taking $\zeta = \zeta' = \zeta_n$, and then integrating, we also have

$$\langle\dot{\Psi}(\zeta_m)|\dot{\Psi}(\zeta_n)\rangle = \mathrm{i}\dot{a}(\zeta_n)\ddot{a}(\zeta_n)(1-\rho^2\zeta_n^{-2})\delta_{mn} + \mathrm{i}2\dot{a}(\zeta_n)^2\rho^2\zeta_n^{-3}\delta_{mn}. \tag{17}$$

Due to the properties of the reduction transformation, it is unnecessary to discuss the inner products involving $\tilde{\Psi}(x,\zeta)$, etc. Equations (15)–(17) are the orthogonality relations we need.

5. The expansion of the unity

Since the above squared Jost functions shall be shown later to form a complete set, then a state $|f\rangle$ can be expanded as

$$|f\rangle = \frac{1}{2\pi}\int_{-\infty}^{\infty} \mathrm{d}\zeta\; f(\zeta)|\Psi(\zeta)\rangle + \sum_{n=1}^{N}\{f_n|\Psi(\zeta_n)\rangle + g_n|\dot{\Psi}(\zeta_n)\rangle\}. \tag{18}$$

By using the orthogonality relations, $f(\zeta)$, f_n and g_n can be expressed in forms of linear combinations of $\langle\Psi(\zeta)|f\rangle$, $\langle\Psi(\zeta_n)|f\rangle$ and $\langle\dot{\Psi}(\zeta_n)|f\rangle$. Substitution of these expressions into (18) yields the expansion of the unity,

$$\begin{aligned}\delta(x-y) = {} & \frac{1}{2\pi}\int_{-\infty}^{\infty}\mathrm{d}\zeta\,\frac{1}{(1-\rho^2\zeta^{-2})a(\zeta)^2}\Psi(x,\zeta)\Phi(y,\zeta)^T(\mathrm{i}\sigma_2)\\ & -\mathrm{i}\sum_{n=1}^{N}\frac{1}{(1-\rho^2\zeta_n^{-2})\dot{a}(\zeta_n)^2}\left\{\dot{\Psi}(x,\zeta_n)\Phi(y,\zeta_n)^T + \Psi(x,\zeta_n)\dot{\Phi}(y,\zeta_n)^T\right\}(\mathrm{i}\sigma_2)\\ & +\mathrm{i}\sum_{n=1}^{N}\left\{\frac{2\rho^2\zeta_n^{-3}}{(1-\rho^2\zeta_n^{-2})^2\dot{a}(\zeta_n)^2} + \frac{\ddot{a}(\zeta_n)}{(1-\rho^2\zeta_n^{-2})\dot{a}(\zeta_n)^3}\right\}\Psi(x,\zeta_n)\Phi(y,\zeta_n)^T(\mathrm{i}\sigma_2)\end{aligned} \tag{19}$$

where, as noted above, ζ should be considered as $\zeta + \mathrm{i}0$, and hence the factor $(1-\rho^2\zeta^{-2})^{-1}$ in the integrand should be replaced by $\zeta^2/[(\zeta+\mathrm{i}0)^2-\rho^2]$. Its poles $\zeta = \pm\rho - \mathrm{i}0$ are on the lower half-plane.

6. Proof of the completeness

To prove (19) it is necessary to show the completeness of the squared Jost functions. For the potential u, we have the Marchenko equation and a complete set of the usual Jost functions,

$$\begin{aligned}&\frac{1}{4\pi}\int_{-\infty}^{\infty}\tilde{\psi}(x,\zeta)\psi(y,\zeta)^T\sigma_1\,\mathrm{d}\zeta + \frac{1}{4\pi}\int_{-\infty}^{\infty} r(\zeta)\psi(x,\zeta)\psi(y,\zeta)^T\sigma_1\,\mathrm{d}\zeta\\ &\qquad -\mathrm{i}\tfrac{1}{2}\sum_{n=1}^{N}c_n\zeta_n\psi(x,\zeta_n)\psi(y,\zeta_n)^T\sigma_1 = \delta(x-y).\end{aligned} \tag{20}$$

For the other potential u', we have

$$\begin{aligned}&\frac{1}{4\pi}\int_{-\infty}^{\infty}\tilde{\psi}'(x,\zeta)\psi'(y,\zeta)^T\sigma_1\,\mathrm{d}\zeta + \frac{1}{4\pi}\int_{-\infty}^{\infty} r'(\zeta)\psi'(x,\zeta)\psi'(y,\zeta)^T\sigma_1\,\mathrm{d}\zeta\\ &\qquad -\mathrm{i}\tfrac{1}{2}\sum_{n'=1}^{N'}c'_{n'}\zeta'_{n'}\psi'(x,\zeta'_{n'})\psi'(y,\zeta'_{n'})^T\sigma_1 = \delta(x-y).\end{aligned} \tag{21}$$

We now consider a kernel $M(x,y)$ that transforms the Jost function $\psi(x,\zeta)$ to the Jost function $\psi'(x,\zeta)$ such that

$$\psi'(x,\zeta) = \psi(x,\zeta) + \int_x^{\infty}\mathrm{d}y\,M(x,y)\psi(y,\zeta) \tag{22}$$

and the inverse kernel $N(x,y)$

$$\psi(x,\zeta) = \psi'(x,\zeta) + \int_x^{\infty}\mathrm{d}y\,N(x,y)\psi'(y,\zeta). \tag{23}$$

By a procedure similar to that for the NLS equation and for the KdV equation [16, 21, 22], substituting (22) into (20) we obtain

$$\frac{1}{4\pi}\int_{-\infty}^{\infty}\tilde{\psi}'(x,\zeta)\psi(y,\zeta)^T\sigma_1\,\mathrm{d}\zeta+\frac{1}{4\pi}\int_{-\infty}^{\infty}r(\zeta)\psi'(x,\zeta)\psi(y,\zeta)^T\sigma_1\,\mathrm{d}\zeta$$
$$-\mathrm{i}\tfrac{1}{2}\sum_{n=1}^{N}c_n\zeta_n\psi'(x,\zeta_n)\psi(y,\zeta_n)^T\sigma_1=\delta(x-y)+\int_x^{\infty}\mathrm{d}s\,M(x,s)\delta(s-y). \tag{24}$$

Similarly, we have

$$\frac{1}{4\pi}\int_{-\infty}^{\infty}\tilde{\psi}'(x,\zeta)\psi(y,\zeta)^T\sigma_1\,\mathrm{d}\zeta+\frac{1}{4\pi}\int_{-\infty}^{\infty}r'(\zeta)\psi'(x,\zeta)\psi(y,\zeta)^T\sigma_1\,\mathrm{d}\zeta$$
$$-\mathrm{i}\tfrac{1}{2}\sum_{n'=1}^{N'}c'_{n'}\zeta'_{n'}\psi'(x,\zeta'_{n'})\psi(y,\zeta'_{n'})^T\sigma_1$$
$$=\delta(x-y)+\int_y^{\infty}\mathrm{d}s\,\delta(x-s)\sigma_1N(s,y)^T\sigma_1. \tag{25}$$

Subtracting (24) from (25), we obtain in the case of $y>x$,

$$\frac{1}{4\pi}\int_{-\infty}^{\infty}\{r'(\zeta)-r(\zeta)\}\psi'(x,\zeta)\psi(y,\zeta)^T\sigma_1\,\mathrm{d}\zeta-\mathrm{i}\tfrac{1}{2}\sum_{n'=1}^{N'}c'_{n'}\zeta'_{n'}\psi'(x,\zeta'_{n'})\psi(y,\zeta'_{n'})^T\sigma_1$$
$$+\mathrm{i}\tfrac{1}{2}\sum_{n=1}^{N}c_n\zeta_n\psi'(x,\zeta_n)\psi(y,\zeta_n)^T\sigma_1=-M(x,y). \tag{26}$$

By using (22) again, we obtain the generalized Marchenko equation

$$M(x,y)+\Omega(x,y)+\int_x^{\infty}\mathrm{d}z\,M(x,z)\,\Omega(z,y)=0\qquad y\geqslant x \tag{27}$$

where

$$\Omega(x,y)=\tfrac{1}{2}\mathrm{i}\sum_n c_n\zeta_n\psi(x,\zeta_n)\psi(y,\zeta_n)^T\sigma_1-\tfrac{1}{2}\mathrm{i}\sum_{n'}c'_{n'}\zeta'_{n'}\psi(x,\zeta'_{n'})\psi(y,\zeta'_{n'})^T\sigma_1$$
$$+\frac{1}{4\pi}\int_{-\infty}^{\infty}\mathrm{d}\zeta\,\{r'(\zeta)-r(\zeta)\}\psi(x,\zeta)\psi(y,\zeta)^T\sigma_1. \tag{28}$$

We find

$$u(x)-u'(x)=-2M(x,x)_{12}\qquad\overline{u(x)}-\overline{u'(x)}=-2M(x,x)_{21}. \tag{29}$$

When $\delta u(x)=u(x)-u'(x)$ is small, we obtain

$$M(x,y)\approx-\Omega(x,y) \tag{30}$$

and

$$\delta\widehat{u(x)}=-\tfrac{1}{2}\mathrm{i}\sum_n[\delta(c_n\zeta_n)\Psi(x,\zeta_n)+c_n\zeta_n\delta\zeta_n\dot{\Psi}(x,\zeta_n)]+\frac{1}{4\pi}\int_{-\infty}^{\infty}\mathrm{d}\zeta\,\delta r(\zeta)\Psi(x,\zeta) \tag{31}$$

where

$$\delta\widehat{u(x)}\equiv\begin{pmatrix}\delta u(x)\\ \delta\overline{u(x)}\end{pmatrix}. \tag{32}$$

Equation (31) implies that the squared Jost functions

$$\frac{\delta\widehat{u(x)}}{\delta r(\zeta)}\propto\Psi(x,\zeta)\qquad\frac{\delta\widehat{u(x)}}{\delta(c_n\zeta_n)}\propto\Psi(x,\zeta_n)\qquad\frac{\delta\widehat{u(x)}}{\delta\zeta_n}\propto\dot{\Psi}(x,\zeta_n) \tag{33}$$

form a complete set.

7. Secularity conditions

Suppose the unknown function $|\boldsymbol{q}\rangle$ in (6) can be expanded as $|f\rangle$ in (18). Then substituting the expansion into (6) and evaluating the inner product of $\langle\Psi(\zeta_n)|$ with the resulted equation, we obtain

$$\mathrm{i}(1-\rho^2\zeta_n^{-2})\dot{a}(\zeta_n)^2\{\mathrm{i}g_{nt}+4\kappa_n\lambda_n g_n\}=\langle\Psi(\zeta_n)|\boldsymbol{R}\rangle \tag{34}$$

etc. Since the initial values of g_n vanish, its value may go to infinity unless the right-hand side of (34) vanishes. Hence we demand it to vanish,

$$\langle\Psi(\zeta_n)|\boldsymbol{R}\rangle=0. \tag{35}$$

A similar discussion for the inner product to $\langle\dot{\Psi}(\zeta_n)|$ demands

$$\langle\dot{\Psi}(\zeta_n)|\boldsymbol{R}\rangle=0. \tag{36}$$

These are the secularity conditions. From them the adiabatic soliton solution, for example, the time dependence of the parameters characterizing the soliton solution of the order of ϵ can be determined.

After determining the adiabatic solution, the inner product of $\langle\Psi(\zeta)|$ with the above resulting equation is

$$\mathrm{i}(1-\rho^2\zeta^{-2})a(\zeta)^2\{\mathrm{i}f_t(\zeta)+4\kappa\lambda f(\zeta)\}=\langle\Psi(\zeta)|\boldsymbol{R}\rangle \tag{37}$$

which can be solved since the terms on the right-hand side has been determined.

The secularity conditions (35) and (36) can be easily reduced to the one-soliton case, which has already been given in [16]. Shown in the case of one soliton, equation (36) corresponds to two equations, one containing limited terms and the other containing terms diverging with the length of the system [16]. So for the N-soliton case, we just have $3N$ independent equations for $3N$ independent parameters. No difficulty of self-consistency found in the method using conservation laws [13] appears. For specific problems, such as when $f(\zeta)$ in (37) is solved, one can find that it is singular at $\zeta=\pm\rho$. However, such singularities cause no secular behaviour of q because $\pm\rho$ should be treated as $\pm\rho-\mathrm{i}0^+$ in order to satisfy the completeness (see also [16]). Therefore, there is no additional secularity conditions at $\zeta=\pm\rho$ suggested by [14].

Acknowledgments

One of authors, NNH acknowledges the sponsorship by National Science Council of the Republic of China and National Chiao Tung University, Hsinchu, Taiwan, Republic of China. This work was supported by National Science Council of the Republic of China under grant NSC87-2215-E009-014.

References

[1] Haus H A and Wong W S 1996 *Rev. Mod. Phys.* **68** 423
[2] Hasegawa A and Kodama K 1995 *Solitons in Optical Communications* (Oxford: Oxford University Press)
[3] Kivshar Y S 1993 Dark solitons in nonlinear optics *IEEE J. Quantum Electron.* **29** 250–64
[4] Zakharov V E and Shabat A B 1973 Interaction between solitons in a stable medium *Sov. Phys.–JETP* **37** 923–30
[5] Faddeev L D and Takhtajan L A 1987 *Hamiltonian Methods in the Theory of Solitons* (Berlin: Springer)
[6] Kaup D J and Newell A C 1978 Solitons as particles, oscillators, and in slowly changing media: a singular perturbation theory *Proc. R. Soc.* A **361** 413–46
[7] Karpman V I 1979 Soliton evolution in the presence of perturbation *Phys. Scr.* **20** 462–78
[8] Giannini J A and Joseph R J 1990 The propagation of bright and dark solitons in lossy optical fibers *IEEE J. Quantum Electron.* **26** 2109

[9] Lisak M Anderson D and Malomed B A 1991 Dissipative damping of dark solitons in optical fibers *Opt. Lett.* **16** 1936–7
[10] Kivshar Y S Perturbation-induced dynamics of small-amplitude dark optical solitons *Opt. Lett.* **15** 1273–5
[11] Kivshar Y S and Yang X 1994 Perturbation-induced dynamics of dark solitons *Phys. Rev.* E **49** 1657–70
[12] Ikeda H, Matsumoto M and Hasegawa A 1995 Transmission control of dark solitons by means of nonlinear gain *Opt. Lett.* **20** 1113–5
[13] Burtsev S and Camassa R 1997 Nonadiabatic dynamics of dark solitons *J. Opt. Soc. Am.* B **14** 1782–7
[14] Konotop V V and Vekslerchik V E 1994 Direct perturbation theory for dark solitons *Phys. Rev.* E **49** 2397–407
[15] Kaup D J 1976 Closure of the squared Zakharov–Shabat eigenstates *J. Math. Anal. Appl.* **54** 849–64
[16] Chen X-J, Chen Z-D and Huang N-N 1998 A direct perturbation theory for dark solitons based on a complete set of the squared Jost solutions *J. Phys. A: Math. Gen.* **31** 6929–47
[17] Chen X-J and Chen Z-D 1998 Effects of Nonlinear gain on dark solitons *IEEE J. Quantum Electron.* **34** 1308–11
[18] Chen X-J and Chen Z-D 1998 Dark optical solitons on influence of the self-steepening term *Chinese Phys. Lett.* **15** 504–6
[19] Chen X-J and Chen Z-D 1998 Raman blueshift of optical dark solitons *J. Opt. Soc. Am.* B **15** 2738–41
[20] Chen X-J, Zhang J-Q and Wang Y-Z 1999 Perturbed spatial dark solitons *Acta Phys. Sinica* **8** 284–7 (overseas edition)
[21] Moses H E 1977 *J. Math. Phys.* **18** 2243
[22] Dodd R K and Bullough R K 1979 The generalized Marchenko equation and the canonical structure of the AKNS-ZS inverse method *Phys. Scr.* **20** 514–30
[23] Deift P and Trubowitz E 1979 Inverse scattering on the line *Commun. Pure. Appl. Math.* **32** 121–251